

# Solve the Jump

## Answer key

Every answer with its work, computed in code from the same numbers as the student workbook. Rounding: answers follow the precision of the given numbers; small differences in the last digit from rounding along the way are fine.

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**Solve it. Don't try it.**

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# The Ramp Kit

- 1** ★ ROOKIE Clue: a training video runs at 60 frames per second. The wheel crosses a 10-foot chalk box in 10 frames. How fast is it going, in feet per second and in mph? (1 mph = 1.467 ft/s.)

**Answer.** 60 ft/s, about 40.9 mph.

- $10 \text{ frames} \div 60 \text{ frames per second} = 1/6 \text{ of a second.}$
- $10 \text{ ft} \div (1/6 \text{ s}) = 60 \text{ ft/s.}$
- $60 \div 1.467 = 40.9 \text{ mph.}$

- 2** ★ ROOKIE The table jumps from 26 ft to 33 ft. Estimate the speed for a 30-ft gap from the table, then check it with the formula.

**Answer.** About 26.4 mph from the table; the formula gives 26.4 mph. Reading between two nearby rows of the table works well.

- 30 ft is  $4/7$  of the way from 26 to 33 ft, so about  $24.6 + 4/7 \times 3.1 \approx 26.4 \text{ mph.}$
- $30 \text{ ft} = 9.14 \text{ m} \rightarrow \sqrt{(9.14 \times 9.8 \div 0.643)} = 11.81 \text{ m/s} = 26.4 \text{ mph.}$

- 3** ★★ PRO Clue: the launch ramp rises 5 ft over a 13.7 ft run along the ground. What angle is it?

**Answer.** About  $20^\circ$ : the kit's angle.

- $\tan(\text{angle}) = \text{rise} \div \text{run} = 5 \div 13.7 = 0.365.$
- $\text{angle} = \tan^{-1}(0.365) = 20.1^\circ.$

- 4** ★★ PRO The speed ladder goes 25, 30, 35, then 40 mph. What gap goes with each step on the kit? When the speed doubles, what happens to the gap?

**Answer.** 25 mph  $\rightarrow$  27 ft; 30 mph  $\rightarrow$  39 ft; 35 mph  $\rightarrow$  53 ft; 40 mph  $\rightarrow$  69 ft. The gap grows with speed squared, so double the speed means four times the gap.

- $25 \text{ mph} = 11.18 \text{ m/s} \rightarrow 11.18^2 \times 0.643 \div 9.8 = 8.19 \text{ m} = 26.9 \text{ ft.}$
- $30 \text{ mph} = 13.41 \text{ m/s} \rightarrow 13.41^2 \times 0.643 \div 9.8 = 11.80 \text{ m} = 38.7 \text{ ft.}$
- $35 \text{ mph} = 15.65 \text{ m/s} \rightarrow 15.65^2 \times 0.643 \div 9.8 = 16.06 \text{ m} = 52.7 \text{ ft.}$
- $40 \text{ mph} = 17.88 \text{ m/s} \rightarrow 17.88^2 \times 0.643 \div 9.8 = 20.97 \text{ m} = 68.8 \text{ ft.}$

- 5** ★★★ ENGINEER Engineer: rider and wheel together weigh about 275 lb (125 kg). Holding speed up a ramp face takes  $\text{mass} \times g \times \text{speed} \times \sin(\text{angle})$  watts, plus the power to push through the air ( $\frac{1}{2} \times 1.2 \times 0.6 \times \text{speed}^3$ ). Can the wheel hold 40 mph up the  $20^\circ$  face if it can give 8.4 kW and keep its reserve? What's the fastest it can hold? What about a  $15^\circ$  face at 40 mph?

**Answer.** No. 40 mph on  $20^\circ$  takes about 9.55 kW. The most it can hold on  $20^\circ$  is about 36.5 mph. A  $15^\circ$  face at 40 mph takes about 7.73 kW, which fits.

- $20^\circ \text{ at } 40 \text{ mph: } 125 \times 9.8 \times 17.88 \times 0.342 = 7,492 \text{ W, plus drag } 2,058 \text{ W} = 9,550 \text{ W.}$
- Fastest on  $20^\circ$  with 8,400 W: try speeds until the total equals 8,400 W  $\rightarrow$  about  $16.32 \text{ m/s} = 36.5 \text{ mph.}$
- $15^\circ \text{ at } 40 \text{ mph: } 5,669 + 2,058 = 7,728 \text{ W.}$

### THE ENGINEER'S QUESTION

**"Same blueprint, same angle, same height, at every venue."**

**What to look for:** Changing one thing at a time is how you learn what caused a result. With the kit fixed, the only thing left to vary is the speed at the lip, and that's the one thing the rider trains.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# Take Style

- 1** ★ ROOKIE Using the same 2,000 Wh battery, how many Wh per mile would the wheel have to use to go 10 miles farther than it does at 30 Wh per mile? What percent saving is that?

**Answer.** 26.1 Wh per mile, a saving of 3.9 Wh per mile (13%).

- New range =  $66.7 + 10 = 76.7$  miles.
- Energy per mile =  $2,000 \div 76.7 = 26.1$  Wh.
- Saving =  $30 - 26.1 = 3.9$  Wh, which is 13% of 30.

- 2** ★★ PRO Find the drag force at 25 mph (11.18 m/s) facing forward and standing side-on. Air density is  $1.2 \text{ kg/m}^3$  and the drag coefficient is 1.0. The rider's frontal area is  $0.55 \text{ m}^2$  facing forward and  $0.40 \text{ m}^2$  side-on.

**Answer.** About 41.2 N facing forward and 30.0 N side-on.

- Forward:  $\frac{1}{2} \times 1.2 \times 1.0 \times 0.55 \times 11.18^2 = 41.2 \text{ N}$ .
- Side-on:  $\frac{1}{2} \times 1.2 \times 1.0 \times 0.40 \times 11.18^2 = 30.0 \text{ N}$ .

- 3** ★★ PRO The energy to push through the air for one mile is the drag force times the distance (1 mile = 1,609 m). Find it for both stances in Wh (divide joules by 3,600). How much does side-on save per mile, and is that enough for problem 1?

**Answer.** About 18.4 Wh forward and 13.4 Wh side-on, a saving of 5.0 Wh per mile. In this model that's more than the 3.9 Wh needed, so yes. The real test finds out whether the drag coefficient stays the same when you turn.

- Forward:  $41.2 \text{ N} \times 1,609 \text{ m} = 66,334 \text{ J} \div 3,600 = 18.4 \text{ Wh}$ .
- Side-on:  $30.0 \text{ N} \times 1,609 \text{ m} = 48,243 \text{ J} \div 3,600 = 13.4 \text{ Wh}$ .
- Saving = 5.0 Wh per mile.

- 4** ★ ROOKIE List five things that must stay the same in both runs for the test to be fair.

**Answer.** Any five of: the same wheel and battery, tire pressure, loop, speed, rider weight and clothing, temperature, wind (or runs in both directions), and starting charge.

## THE ENGINEER'S QUESTION

**"The result gets published even if the stance loses."**

**What to look for:** A test you'd hide if it went the wrong way isn't a test. Publishing a loss is what makes people trust a win.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# The Empty Bus

- 1** **★★ PRO** The jump. The bus is about 3.2 m (10.5 ft) tall. The season's 20° ramp kit sits on 3.0 m (10 ft) towers, and the wheel should clear the roof by 0.5 m at the top of the arc. What speed does it take at the lip?

**Answer.** 10.8 m/s, about 24.2 mph.

- Height above the lip =  $3.2 + 0.5 - 3.0 = 0.7$  m.
- Upward speed =  $\sqrt{(2 \times 9.8 \times 0.7)} = 3.70$  m/s.
- Speed at the lip =  $3.70 \div \sin(20^\circ) = 3.70 \div 0.342 = 10.83$  m/s = 24.2 mph.

- 2** **★★★ ENGINEER** With identical ramps at the same height, the top of the arc is exactly halfway across. At the speed from problem 1, how wide is the gap between the lips? The bus is 2.6 m wide and parked in the middle. How much room is there over the roof's edges?

**Answer.** The gap is about 7.7 m (25 ft), with the top of the arc 3.8 m from each lip. Over the roof's edges the wheel still clears by about 42 cm.

- Gap =  $10.83^2 \times \sin(40^\circ) \div 9.8 = 7.69$  m; halfway = 3.85 m.
- The roof's near edge is 1.3 m before the middle:  $x = 2.55$  m.
- Height there =  $x \times \tan 20^\circ - 9.8 \times x^2 \div (2 \times 10.83^2 \times \cos^2 20^\circ) = 0.62$  m above the lip.
- Clearance =  $3.0 + 0.62 - 3.2 = 0.42$  m.

- 3** **★ ROOKIE** The bus. Using \$13.44 a mile, how many riders does a bus need for each rider's mile to cost under 50 cents? Under \$1?

**Answer.** 27 riders for under 50 cents, 14 riders for under \$1.

- $13.44 \div 0.50 = 26.88$ , so 27 riders.
- $13.44 \div 1.00 = 13.44$ , so 14 riders.

- 4** **★★ PRO** Now the Lamborghini. Suppose a chauffeured Urus works like this: it costs \$281,000, is driven 45,000 miles a year for 5 years, then sells for \$50,000. It gets 20 mpg with gas at \$3.20. Insurance is \$12,000 a year. Repairs and tires are 50 cents a mile. The driver costs \$40 an hour, and city traffic averages 14 mph. What does it cost per mile? Per rider-mile with 4 riders?

**Answer.** About \$4.81 a mile, or \$1.20 per rider-mile with 4 riders, less than the average bus's \$2.10.

- Car:  $(281,000 - 50,000) \div 225,000$  miles = \$1.03 a mile.
- Gas:  $3.20 \div 20 = \$0.16$ .
- Insurance:  $12,000 \div 45,000 = \$0.27$ .
- Repairs: \$0.50.
- Driver:  $40 \div 14 = \$2.86$ .
- Total = \$4.81 a mile (adding the rounded parts gives \$4.82; either is fine).  $\div 4$  riders = \$1.20.

- 5** **★★ PRO** How many riders does the bus need before each one rides for less than in the full SUV? Then rush hour: 60 people at one stop. What does it cost per mile to move them in SUVs, 4 to a car, compared with one bus?

**Answer.** 12 riders (11.2 to break even). At rush hour it takes 15 SUVs and 15 drivers: about \$72 a mile against \$13.44 for one bus, about 5.4 times more.

- $13.44 \div 1.20 = 11.2$ , so 12 riders.
- $60 \div 4 = 15$  SUVs;  $15 \times \$4.81 = \$72.16$  a mile.
- $72.16 \div 13.44 = 5.4$ .

- 6** **★ ROOKIE** So why do cities run big buses all day? Give two reasons. Then name one thing a city could do when a route runs nearly empty.

**Answer.** Reasons: rush hour needs the room, and the buses and drivers have to exist for rush hour anyway; one driver can move 60 people; big buses use less street per rider. Ideas: smaller vehicles or on-demand vans at quiet times, fixing routes, or running often and on time so more people ride.

### THE ENGINEER'S QUESTION

**"Landed over a bus-sized frame at ground level first."**

**What to look for:** A frame the size of the bus proves the speed, the lip position and the landing where a miss costs nothing. The real bus only goes between the ramps once the numbers repeat.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# 79 in One

- 1** ★ ROOKIE Five checkpoints, A to E, are this many miles apart: A–B 4, A–C 7, A–D 6, A–E 3, B–C 3, B–D 8, B–E 5, C–D 4, C–E 6, D–E 5. Start and finish at A and visit each checkpoint once. There are 12 different loops. Find the shortest.

**Answer.** A–B–C–D–E–A, 19 miles (or the same loop backwards). The longest loop is A–C–E–B–D–A at 32 miles.

- A–B–C–D–E–A: 19 miles
- A–D–C–B–E–A: 21 miles
- A–B–C–E–D–A: 24 miles
- A–B–D–C–E–A: 25 miles
- A–B–E–C–D–A: 25 miles
- A–B–E–D–C–A: 25 miles
- A–C–B–D–E–A: 26 miles
- A–C–B–E–D–A: 26 miles
- A–D–B–C–E–A: 26 miles
- A–C–D–B–E–A: 27 miles
- A–B–D–E–C–A: 30 miles
- A–C–E–B–D–A: 32 miles

- 2** ★ ROOKIE Suppose the full 79-neighborhood route is 120 miles at 30 Wh per mile. How much energy does it need? Keeping a 20% reserve (1,600 Wh usable per charge) and starting full, how many charging stops does it take?

**Answer.** 3,600 Wh. 2 charging stops.

- $120 \times 30 = 3,600$  Wh.
- Starting full plus 1 stop gives  $2 \times 1,600 = 3,200$  Wh, not enough.
- Starting full plus 2 stops gives  $3 \times 1,600 = 4,800$  Wh. Enough, so 2 stops.

- 3** ★★ PRO Sunrise to midnight is 17 hours. The wheel averages 14 mph while moving. Each of the 79 checkpoints takes 3 minutes for its photo. Each charging stop puts back 1,600 Wh from an 800-watt charger. Does the day fit, and how much time is left over?

**Answer.** Yes, barely. 16.5 hours of the 17, with about 29 minutes to spare.

- Riding:  $120 \div 14 = 8.57$  hours.
- Checkpoints:  $79 \times 3 = 237$  minutes = 3.95 hours.
- Charging:  $2 \times (1,600 \div 800) = 4$  hours.
- Total = 16.52 hours.  $17 - 16.52 = 0.48$  hours  $\approx$  29 minutes.

## THE ENGINEER'S QUESTION

**"Daylight for the worst pavement."**

**What to look for:** Potholes and cracks are easy to see in daylight and easy to miss at night. The route plan puts the roughest streets in the brightest hours.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# The Keg Line

**1** ★★ PRO Twelve kegs off the same 20° ramps. What speed at the lip?

**Answer.** 8.62 m/s, about 19.3 mph.

- $12 \times 16 \text{ in} = 4.88 \text{ m}$ .
- $\sqrt{(4.88 \times 9.8 \div 0.643)} = 8.62 \text{ m/s} = 19.3 \text{ mph}$ .

**2** ★★ PRO Ten kegs again, but with 30° ramps. What speed now? Give one reason the rider might still choose 20°.

**Answer.** 6.78 m/s, about 15.2 mph. A steeper ramp throws the wheel higher, so the fall is longer and the landing is harder.

- $\sin(60^\circ) = 0.866$ .
- $\sqrt{(4.06 \times 9.8 \div 0.866)} = 6.78 \text{ m/s} = 15.2 \text{ mph}$ .

**3** ★★ PRO How long is the wheel in the air in the worked example? Time in the air =  $2 \times \text{speed} \times \sin(\text{angle}) \div g$ .

**Answer.** About 0.55 seconds.

- $2 \times 7.87 \times \sin(20^\circ) \div 9.8 = 2 \times 7.87 \times 0.342 \div 9.8 = 0.55 \text{ s}$ .

**4** ★★★ ENGINEER The rider arrives 1 mph slow on the ten-keg jump at 20°. How far does the wheel go, and how short does it land?

**Answer.** 3.62 m (11.9 ft). That's 0.45 m short, about 1.1 kegs. Range goes with speed squared, so 5.7% less speed costs about 11% of the distance.

- Speed =  $7.87 - 0.45 = 7.42 \text{ m/s}$ .
- Distance =  $7.42^2 \times 0.643 \div 9.8 = 3.62 \text{ m}$ .

## THE ENGINEER'S QUESTION

**"The count only goes up after a clean landing."**

**What to look for:** Each clean landing proves the math and the ramps before any more risk is added. Problem 4 shows how fast a small mistake becomes a short landing.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# Jump the Zamboni

- 1 ★★ PRO** Suppose the machine is 2.0 m tall and the kit's lips are 1.5 m (5 ft) off the ice. The wheel should clear the machine by 0.3 m at the top of the arc. How high above the lip must the arc peak, and what speed does that take at the lip?

**Answer.** 0.8 m above the lip, which takes 11.6 m/s, about 25.9 mph.

- $2.0 + 0.3 - 1.5 = 0.8$  m.
- Upward speed =  $\sqrt{(2 \times 9.8 \times 0.8)} = 3.96$  m/s.
- Speed at the lip =  $3.96 \div \sin(20^\circ) = 11.58$  m/s = 25.9 mph.

- 2 ★★★ ENGINEER** The quickest a tire can speed you up is about (friction coefficient  $\times$  g). Say plain rubber on ice is 0.1 and a studded tire is 0.3. The run-up needed =  $\text{speed}^2 \div (2 \times \text{acceleration})$ . The machine sits at center ice, about 30 m from the end boards. Which tire can reach the speed from problem 1 in time?

**Answer.** Plain rubber needs about 68 m, which doesn't fit. Studs need about 23 m, which does.

- Rubber:  $a = 0.1 \times 9.8 = 0.98$  m/s<sup>2</sup>;  $11.58^2 \div (2 \times 0.98) = 68.4$  m.
- Studs:  $a = 0.3 \times 9.8 = 2.94$  m/s<sup>2</sup>;  $11.58^2 \div (2 \times 2.94) = 22.8$  m.

- 3 ★★★ ENGINEER** The rider arrives 1 mph slow. Does the wheel still clear the machine, and by how much?

**Answer.** Yes, by about 0.24 m instead of 0.3 m.

- Speed =  $11.58 - 0.45 = 11.13$  m/s; upward part = 3.81 m/s.
- Peak =  $3.81^2 \div 19.6 = 0.74$  m above the lip, 2.24 m off the ice.
- Clearance =  $2.24 - 2.0 = 0.24$  m.

- 4 ★★ PRO** Where does the machine go? With identical ramps the top of the arc is halfway across. At the speed from problem 1, how wide is the gap between the lips, and how far from the launch lip is the top of the arc?

**Answer.** The gap is about 8.8 m (29 ft), and the top of the arc is 4.4 m from the launch lip. The middle of the machine goes there.

- Gap =  $11.58^2 \times \sin(40^\circ) \div 9.8 = 8.79$  m.
- Halfway = 4.40 m.

## THE ENGINEER'S QUESTION

**"The studded tire is tested for grip before any ramp goes down."**

**What to look for:** Problem 2 shows the whole jump depends on reaching takeoff speed on ice. If the grip number is wrong, the rider arrives slow, and problem 3 shows what slow does to the clearance. Test the assumption first.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# The Parade

- 1** ★ ROOKIE A rider's balance point is about 1.0 m above where the tire touches the ground. What's the fall time? If the wheel's controller corrects 1,000 times a second, how many corrections fit inside it?

**Answer.** About 0.32 s, which fits about 319 corrections.

- $\sqrt{(1.0 \div 9.8)} = 0.319$  s.
- $0.319 \times 1,000 = 319$  corrections.

- 2** ★★ PRO Side to side, the rider stays up by steering. The sideways push a turn gives you is  $\text{speed}^2 \div \text{turn radius}$ . Compare 3 mph and 12 mph through the same 3 m turn.

**Answer.**  $0.60 \text{ m/s}^2$  at 3 mph and  $9.59 \text{ m/s}^2$  at 12 mph, 16 times more. Four times the speed gives sixteen times the push, so at walking pace steering barely helps and the rider has to correct sooner.

- $3 \text{ mph} = 1.34 \text{ m/s}$ ;  $1.34^2 \div 3 = 0.60 \text{ m/s}^2$ .
- $12 \text{ mph} = 5.36 \text{ m/s}$ ;  $5.36^2 \div 3 = 9.59 \text{ m/s}^2$ .

- 3** ★ ROOKIE Experiment: balance a broomstick on your palm, then a 30 cm ruler, five tries each, and time each try. Average them. Which lasted longer, and by about how much does the fall-time formula say it should?

**Answer.** The broomstick should last longer. The formula predicts about  $\sqrt{(1.5 \div 0.3)} = 2.2$  times longer fall time. Real results vary with skill, which is why you take five tries.

## THE ENGINEER'S QUESTION

**"No kids inside the stunt zone."**

**What to look for:** Kids get pulled toward the action and move fast and unpredictably. The math in this chapter keeps the rider up; it can't account for someone stepping in front of him.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# The Bend

**1** ★★ PRO The same 50 m turn at 20 mph. What's the lean?

**Answer.** About  $9.3^\circ$ . Two-thirds the speed, less than half the lean.

- $20 \text{ mph} = 8.94 \text{ m/s}$ ;  $8.94^2 \div 490 = 0.163$ ; lean =  $9.3^\circ$ .

**2** ★★★ ENGINEER A tire slides when  $\tan(\text{lean})$  would have to be more than the friction coefficient. With 0.8 on dry pavement, what's the most lean, and the fastest speed around the 50 m turn? With 0.5 on a damp deck?

**Answer.** Dry:  $38.7^\circ$  and 44.3 mph. Damp:  $26.6^\circ$  and 35.0 mph.

- Fastest speed =  $\sqrt{(\text{friction} \times g \times \text{radius})}$ .
- Dry:  $\sqrt{(0.8 \times 9.8 \times 50)} = 19.80 \text{ m/s} = 44.3 \text{ mph}$ .
- Damp:  $\sqrt{(0.5 \times 9.8 \times 50)} = 15.65 \text{ m/s} = 35.0 \text{ mph}$ .

**3** ★★★ ENGINEER Suppose a map shows the bridge's bend turning  $22^\circ$  across 120 m of deck. Radius = arc length  $\div$  angle in radians. Find the radius, then the lean at 30 mph. Is the bend the hard part?

**Answer.** About 313 m, and a lean of only  $3.4^\circ$ . The bend is gentle; the hard parts are the surface, the dark and the speed.

- $22^\circ = 22 \times \pi \div 180 = 0.384 \text{ radians}$ .
- Radius =  $120 \div 0.384 = 312.5 \text{ m}$ .
- $\tan(\text{lean}) = 179.9 \div (312.5 \times 9.8) = 0.0587$ ; lean =  $3.4^\circ$ .

## THE ENGINEER'S QUESTION

**"The deck inspected seam by seam."**

**What to look for:** Problem 2 shows the lean depends on grip. A gap, a bump or a slick patch at speed takes grip away all at once, so every seam gets checked before the run.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# The Spirit

**1** ★★ PRO How much power does it take at 40 mph?

**Answer.** About 2,058 W.

- $40 \text{ mph} = 17.88 \text{ m/s}$ ;  $\frac{1}{2} \times 1.2 \times 0.6 \times 17.88^3 = 2,058 \text{ W}$ .

**2** ★★★ ENGINEER The motor can put 3,000 W into fighting the air, but 30% has to stay in reserve for balance. What's the top speed?

**Answer.** About 18.0 m/s, or 40.3 mph.

- Usable = 70% of 3,000 = 2,100 W.
- Speed =  $(2,100 \div (\frac{1}{2} \times 1.2 \times 0.6))^{(1/3)} = (5,833)^{(1/3)} = 18.00 \text{ m/s} = 40.3 \text{ mph}$ .

**3** ★★★ ENGINEER Tucking cuts the drag area to  $0.45 \text{ m}^2$ . With the same usable power, what's the new top speed? What drag area would it take to gain a full 5 mph over problem 2?

**Answer.** About 44.3 mph. Gaining 5 mph takes a drag area of about  $0.42 \text{ m}^2$ , a 30% cut.

- $(2,100 \div (\frac{1}{2} \times 1.2 \times 0.45))^{(1/3)} = 19.81 \text{ m/s} = 44.3 \text{ mph}$ .
- Target 45.3 mph = 20.24 m/s; area =  $2,100 \div (\frac{1}{2} \times 1.2 \times 20.24^3) = 0.422 \text{ m}^2$ .

**4** ★ ROOKIE With the wind behind him the timing trap reads 44 mph. Against the wind it reads 38 mph. What speed goes in the record, and why run both directions?

**Answer.** 41 mph, the average. Running both ways cancels most of the wind's help, so the record measures the rider and the wheel.

- $(44 + 38) \div 2 = 41 \text{ mph}$ .

## THE ENGINEER'S QUESTION

**"A hard cap on speed that keeps motor power in reserve."**

**What to look for:** The motor is also what keeps the rider from falling forward. Problem 2 shows the cap: run the motor out of power and it can't catch the next lean.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# Art Hill

- 1** ★★ PRO What if the drop is 3.0 m? How much faster is that than 1.5 m?

**Answer.** 7.67 m/s (17.2 mph). Twice the drop is about 1.41 times the speed, because of the square root.

- $\sqrt{(2 \times 9.8 \times 3.0)} = \sqrt{58.8} = 7.67 \text{ m/s.}$

- 2** ★★ PRO The wheel is moving forward at 8.0 m/s and falls 1.5 m. At what angle below horizontal is it moving when it lands?

**Answer.** About 34°. A landing ramp near that angle matches the path.

- $\tan(\text{angle}) = \text{downward} \div \text{forward} = 5.42 \div 8.0 = 0.678$ ; angle = 34.1°.

- 3** ★★★ ENGINEER Land on a 30° down-ramp instead of flat ground. Only the speed into the ramp has to be stopped: (downward  $\times \cos 30^\circ$ ) – (forward  $\times \sin 30^\circ$ ). Find it. What share of the flat-ground hit is left?

**Answer.** 0.70 m/s into the ramp, about 13% of the 5.42 m/s on flat ground. The ramp takes away about 87%.

- $5.42 \times 0.866 - 8.0 \times 0.5 = 4.70 - 4.00 = 0.70 \text{ m/s.}$

- 4** ★★★ ENGINEER Bending the knees gives about 0.30 m to stop in. Average stopping acceleration =  $\text{speed}^2 \div (2 \times \text{distance})$ . Find it in g for the flat landing and the ramp landing.

**Answer.** Flat: about 5.0 g on top of gravity. Ramp: about 0.08 g, almost nothing.

- Flat:  $5.42^2 \div 0.6 = 49.0 \text{ m/s}^2 \div 9.8 = 5.0 \text{ g.}$
- Ramp:  $0.70^2 \div 0.6 = 0.81 \text{ m/s}^2 \div 9.8 = 0.08 \text{ g.}$

## THE ENGINEER'S QUESTION

**"Crowd lines set well back from the landing."**

**What to look for:** Chapter 10 shows every jump has a window of speeds, and a jump that's too fast carries past the end of the landing. The crowd stands beyond where the worst case could reach.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# Center Stage

- 1** ★ ROOKIE Riding 15 mph, he throws forward at 45 mph. What's the ball's speed over the ground?

**Answer.** 60 mph.

- $15 + 45 = 60$  mph.

- 2** ★ ROOKIE The radar on the plaza reads the pitch at 68 mph. The wheel's GPS says it was doing 21 mph. How hard did he actually throw?

**Answer.** 47 mph.

- $68 - 21 = 47$  mph.

- 3** ★ ROOKIE Riding 20 mph forward, he tosses the ball backward at 20 mph. What does the crowd see the ball do? What does the rider see?

**Answer.** The crowd sees it drop almost straight down: its sideways speeds cancel. The rider sees it fly backward away from him at 20 mph.

- Over the ground:  $20 - 20 = 0$  mph sideways, so it just falls.

- 4** ★★ PRO The jump: 35 mph at the lip on the 20° kit. How wide is the gap, how long is the wheel in the air, and how high above the lip does it go?

**Answer.** About 53 ft (16.1 m), 1.09 seconds, and 4.8 ft (1.46 m) above the lip.

- $35 \text{ mph} = 15.65 \text{ m/s}$ .
- $\text{Gap} = 15.65^2 \times \sin 40^\circ \div 9.8 = 16.06 \text{ m}$ .
- $\text{Air time} = 2 \times 15.65 \times \sin 20^\circ \div 9.8 = 1.09 \text{ s}$ .
- $\text{Height} = (15.65 \times \sin 20^\circ)^2 \div 19.6 = 1.46 \text{ m}$ .

- 5** ★★★ ENGINEER The runway. Say the wheel speeds up at 0.3 g and brakes at 0.4 g. How much runway does it take to reach 35 mph? To stop after landing? Add about 15 ft for each ramp and the gap from problem 4. How long does the runway have to be?

**Answer.** About 137 ft to get up to speed and 102 ft to stop. With the ramps and the gap, about 322 ft of runway.

- Speeding up:  $15.65^2 \div (2 \times 2.94) = 41.6 \text{ m} = 137 \text{ ft}$ .
- Stopping:  $15.65^2 \div (2 \times 3.92) = 31.2 \text{ m} = 102 \text{ ft}$ .
- Total =  $137 + 15 + 53 + 15 + 102 \approx 322 \text{ ft}$ .

## THE ENGINEER'S QUESTION

**"Room to stop before the stage."**

**What to look for:** Problem 5 shows the wheel needs about a hundred feet to stop from 35 mph after it lands. If that room isn't there, the jump moves or shrinks, not the other way around.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.

# The Wash Ave Gap

- 1** ★ ROOKIE Every 1 mph moves the landing about 3.4 ft. If the rider hits the lip 3 mph slow, about how far short of 69 ft does he come down? Check it with the gap formula at 37 mph.

**Answer.** About 10 ft short. The formula gives 59 ft at 37 mph.

- $3 \times 3.4 \approx 10$  ft short.
- $37 \text{ mph} = 16.54 \text{ m/s}$ ;  $16.54^2 \times \sin 40^\circ \div 9.8 = 17.94 \text{ m} = 58.9 \text{ ft}$ .

- 2** ★★ PRO The safe landing zone runs from 64 ft to 74 ft past the lip. What speed window does that give?

**Answer.** From 38.6 mph to 41.5 mph, only about  $\pm 1.5$  mph around 40.

- $\text{Speed} = \sqrt{(\text{gap} \times 9.8 \div \sin 40^\circ)}$ .
- $64 \text{ ft} = 19.51 \text{ m} \rightarrow 17.25 \text{ m/s} = 38.6 \text{ mph}$ .
- $74 \text{ ft} = 22.56 \text{ m} \rightarrow 18.54 \text{ m/s} = 41.5 \text{ mph}$ .

- 3** ★★★ ENGINEER How long would the landing zone have to be for a  $\pm 3$  mph window around 40 mph?

**Answer.** From about 59 ft to 80 ft past the lip: a zone about 21 ft long.

- $37 \text{ mph} \rightarrow 58.9 \text{ ft}$ .
- $43 \text{ mph} \rightarrow 79.5 \text{ ft}$ .
- $79.5 - 58.9 = 20.6 \text{ ft}$ .

- 4** ★★★ ENGINEER Design the finale kit. Compare a  $20^\circ$  kit and a  $15^\circ$  kit at 40 mph: the gap, the height above the lip, the air time, and the power to hold 40 mph up the ramp face (275 lb of rider and wheel, drag area  $0.6 \text{ m}^2$ ; see The Ramp Kit). If 8.4 kW is all the wheel can give while keeping its reserve, which do you pick and why?

**Answer.**  $20^\circ$ : 69 ft, 6.3 ft up, 1.25 s, 9.55 kW.  $15^\circ$ : 54 ft, 3.6 ft up, 0.94 s, 7.73 kW. Only the  $15^\circ$  kit holds 40 mph up its face within 8.4 kW. A good answer might also keep  $20^\circ$  and come in faster, letting the ramp slow the wheel to 40 at the lip. Grade the reasoning.

- $\text{Climb power} = 125 \times 9.8 \times 17.88 \times \sin(\text{angle})$ ;  $\text{drag} = \frac{1}{2} \times 1.2 \times 0.6 \times 17.88^3 = 2,058 \text{ W}$ .
- $20^\circ$ :  $7,492 + 2,058 = 9,550 \text{ W}$ .
- $15^\circ$ :  $5,669 + 2,058 = 7,727 \text{ W}$ .

## THE ENGINEER'S QUESTION

**"If any one of those is a no, it doesn't happen."**

**What to look for:** With one attempt, no safeguard can stand in for another. Each one covers a different way to fail, so a missing one leaves that failure uncovered.

Predict the stunt: there's no key. Predictions are scored against the real numbers on the night.